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Lift-Dispersion Control of Spinning Reentry Vehicles

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Gerhard E. Aichinger

Project Officer

FOR THE COMMANDER

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7	that would trim the vehicle at zero angle of attack. A simple control law is
	that would trim the vehicle at zero angle of attack. A simple control law is derived that specifies minimum feedbacks required, and the open- and closed-
	loop vehicle response to impulsive disturbance moments is demonstrated.
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# PREFACE

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#### I. INTRODUCTION

Significant dispersion errors can result from lift variations during the reentry phase of a long-range ballistic reentry vehicle trajectory. The most notable errors result when the roll rate is driven to zero with the presence of trim asymmetries that cause lift. Such errors can be avoided with roll control by maintenance of a nonzero roll rate during reentry. Even with a steady, nonzero roll rate, lift variations caused by ablation shape change and asymmetric boundary-layer transition progression can cause moderate reentry dispersion errors. Because the magnitudes of force and moment asymmetries that can cause significant dispersion are relatively small, and because the epicyclic component of angle-of-attack motion does not produce dispersion, the motion can be controlled by the use of small pitch and yaw control moments to minimize the net transverse dispersion velocity.

In a previous report. 6 a method was formulated for control of crossrange dispersion of a spinning missile flying in untrimmed motion. The missile was assumed to be untrimmed as a result of another control loop that modulates the angle of attack for drag control of range error. 7 In the untrimmed condition, the lift vector precesses in space at a characteristic frequency that can be well above the roll frequency. The control law for this case differs from that for a vehicle flying in trimmed motion, which is the usual condition of a reentry vehicle after the initial angle-of-attack error has converged. In trimmed flight, the vehicle is in lunar motion with a fixed windward meridian, and the lift vector precesses in space at the roll frequency. The control law for such a system is derived in this report, and the closed-loop response of a reentry vehicle to simple disturbances is compared with the open-loop response. It is shown that control should be possible with information derived from conventional strapdown accelerometers and rate gyros. For the special condition in which the roll rate is maintained at 50% of the undamped natural pitch frequency, control can be achieved with information derived solely from lateral accelerometers. However, this might be difficult to implement because of uncertainty in the aerodynamic pitch frequency and critical roll rate, which could outweigh the advantage of fewer sensors.

#### II. ANALYSIS

### A. Control Equations

The equations of motion of a spinning missile in body-fixed coordinates can be written  $^{8}$ 

$$\ddot{\xi} + F\dot{\xi} + G\xi = i(m_t + m_{\delta})$$
 (1)

where

$$\xi = \beta + i\alpha \tag{2}$$

$$\mathbf{F} = \mathbf{v} + \mathbf{i}\mathbf{p} \ (2 - \mathbf{\mu}) \tag{3}$$

$$G = \omega^2 - p^2 (1 - \mu) + ip (\nu - \mu C_{L_{\alpha}}^*)$$
 (4)

and m<sub>t</sub> and m<sub>o</sub> are complex disturbance and control moments, respectively, defined as a ratio-to-pitch moment of inertia. A complex transverse velocity in the cross plane due to lift can be written<sup>4</sup>

$$\Delta V = v + iw = -\frac{L_{\alpha}}{m} \int_{0}^{t} \xi e^{ipt} dt$$
 (5)

where  $L_{\alpha}$  is the lift force derivative and pt is the roll angle, or precession angle of the lift vector for a trimmed vehicle with constant roll rate p. The upper limit of the integral in Eq. (5) is assumed to be sufficiently large to include perturbations in the complex angle of attack that cause dispersion as a result of lift. The transverse velocity is defined with respect to quasi-inertial coordinates normal to the mean flight path. The complex control

moment  $m_{\delta}$  that will minimize the net transverse velocity  $\Delta V$  caused from some trim disturbance moment  $m_{t}$  is defined as follows. If we resolve Eq. (1) into its real and imaginary components and take the Laplace transform with respect to time, we obtain the coupled linear control equations

$$\begin{bmatrix} \mathbf{s}^{2} + \mathbf{v}\mathbf{s} + \mathbf{w}^{2} - \mathbf{p}^{2}(\mathbf{1} - \mu) & \mathbf{p}(2 - \mu) \mathbf{s} + \mathbf{p}\left(\mathbf{v} - \mu \mathbf{C}_{\mathbf{L}_{\alpha}}^{*}\right) \\ -\mathbf{p}(2 - \mu)\mathbf{s} - \mathbf{p}\left(\mathbf{v} - \mu \mathbf{C}_{\mathbf{L}_{\alpha}}^{*}\right) & \mathbf{s}^{2} + \mathbf{v}\mathbf{s}[\mathbf{w}^{2} - \mathbf{p}^{2}(\mathbf{1} - \mu)] \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{m}_{\mathbf{t}} \\ \mathbf{m}_{\mathbf{t}_{\mathbf{z}}} \end{bmatrix} + \begin{bmatrix} \mathbf{m}_{\delta} \\ \mathbf{y} \\ -\mathbf{m}_{\delta} \end{bmatrix}$$
(6)

With the upper limit of the integral in Eq. (5) assumed to be infinitely large, the Laplace transform of the transverse velocity can be written

$$\Delta V(s) = -\frac{L_{\alpha}}{ms} [\beta(s - ip) + i\alpha (s - ip)]$$
 (7)

where  $\alpha(s - ip)$  and  $\beta(s - ip)$  are functions of s - ip in which the complex translation results from multiplication by the exponential in Eq. (5). We assume control moments of the form

$$m_{\delta y} = -a\alpha - b\beta - c\dot{\alpha} - d\dot{\beta}$$
 (8)

$$m_{\delta_{z}} = a_{1} \alpha + b_{1} \beta + c_{1} \dot{\alpha} + d_{1} \dot{\beta}$$
 (9)

which, when substituted in Eq. (6), give the control equations

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} m_{t_y} \\ -m_{t_z} \end{bmatrix}$$
 (10)

where

$$A_{11} = s^{2} + (v + c) s + w^{2} - p^{2} (1 - \mu) + a$$

$$A_{12} = [p(2 - \mu) + d] s + p \left(v - \mu C_{L_{\alpha}}^{*}\right) + b$$

$$A_{21} = [c_{1} - p (2 - \mu)] s - p \left(v - \mu C_{L_{\alpha}}^{*}\right) + a_{1}$$

$$A_{22} = s^{2} + (v + d_{1}) s + w^{2} - p^{2} (1 - \mu) + b_{1}$$

## B. Solution for Impulsive Trim

Consider an impulsive trim moment of the form

$$m_{t_y} = m_* \delta(t), m_{t_z} = 0$$
 (11)

where  $\delta(t)$  is the unit impulse function. The solution to Eq. (10) is then

$$\alpha = \frac{m_*}{\Delta} [s^2 + (d_1 + v) s + w^2 - p^2 + b_1]$$
 (12)

$$\beta = -\frac{m_*}{\Delta} \left[ (c_1 - 2p) s + a_1 - vp \right]$$
 (13)

<sup>\*</sup>The inertia ratio  $\mu$  has been neglected relative to unity for simplicity.

where

$$\Delta = s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0$$
 (14)

$$A_3 = c + d_1 + 2v (15)$$

$$A_2 = 2 (\omega^2 + p^2) + a + b_1 + cd_1 - c_1 d + v^2$$

$$+ v(c + d_1) + 2p(d - c_1)$$
 (16)

$$A_1 = 2v (w^2 + p^2) + (c + d_1) (w^2 - p^2)$$

$$+ (a + b_1) v + b_1 c + ad_1 - c_1 b$$

$$- vp (c_1 - d) - 2p (a_1 - b) - a_1 d$$
 (17)

$$A_0 = (\omega^2 - p^2)^2 + (a + b_1) (\omega^2 - p^2) + ab_1 - a_1b$$

$$+ p (b - a_1) v + p^2 v^2$$
(18)

and the transverse velocity can be written in the form

$$\frac{\Delta V(s)}{-L_{\alpha}m_{*}/m} = \frac{N_{2}s^{2} + N_{1}s + N_{0}}{s(s^{4} + D_{3}s^{3} + D_{2}s^{2} + D_{1}s + D_{0})} = \frac{N_{0}/D_{0}}{s}$$

$$+ \frac{F(s)}{s^{4} + D_{3}s^{3} + D_{2}s^{2} + D_{1}s + D_{0}} \tag{19}$$

It is well known that the epicyclic component of the motion does not cause dispersion.  $^{4,5}$  Hence, only the residue of the pole at the origin is of interest and the net steady-state transverse velocity increment  $\Delta V_{ss}$  due to the impulse is

$$\frac{\Delta V_{ss}}{-L_{\alpha}m_{*}/m} = \frac{N_{o}}{D_{o}}$$

$$= \frac{-a_1 + d_1p + 2vp + i(\omega^2 + b_1 + c_1p - 4p^2)}{A_0 + p^4 + ip^3A_3 - p^2A_2 - ipA_1}$$
(20)

This velocity will be zero for feedbacks that cause the real and imaginary components of the numerator of Eq. (20) to vanish, which requires that

$$-a_1 + d_1 p + 2 vp = 0 (21)$$

$$\omega^2 + b_1 + c_1 p - 4p^2 = 0 (22)$$

For the assumed feedbacks, Eqs. (8) and (9), only feedbacks for  $m_{\delta_Z}$  are required for the disturbance moments assumed in Eq. (11). For system stability, the coefficients of the characteristic equation, Eq. (14), must be positive and

$$A_1 (A_3 A_2 - A_1) - A_3^2 A_0 > 0 (23)$$

It is found from the coefficients, Eqs. (15) through (18), that these conditions will be satisfied with only the feedback gains a and b. Eqs. (21) and (22) for zero dispersion then require that

$$\mathbf{a_i} = 2 \ \mathsf{vp} \tag{24}$$

$$b_1 = 4p^2 - \omega^2 (25)$$

Because the gains defined by Eqs. (24) and (25) require a knowledge of aerodynamic coefficients, it may be desirable to minimize the dependence on aerodynamics by retaining the feedbacks corresponding to the gains  $c_1$  and  $d_1$  in Eqs. (21) and (22). For the special condition when the roll rate is half the pitch frequency  $(p = \omega/2)$ , the gain  $b_1$  is zero, and only a single feedback is required.

## C. Step Trim Change

With a step change in disturbance moment rather than an impulse, i. e.,

$$m_{t_y} = \Delta m_y H(t) \tag{26}$$

where H(t) is the Heaviside unit step function, the solution to Eq. (10) has the form

$$\alpha = \frac{\Delta m_y}{s\Delta} \left[ s^2 + (d_1 + v) s + \omega^2 - p^2 + b_1 \right]$$
 (27)

$$\beta = -\frac{\Delta m_y}{s\Delta} \left[ (c_1 - 2p) s + a_1 - \nu p \right]$$
 (28)

in which the characteristic  $\Delta$  is unchanged from Eq. (14). A comparison of Eqs. (27) and (28) with Eqs. (12) and (13) indicates the only change in dispersion velocity calculated from Eq. (7) is an additional pole at s - ip. This results in a component of transverse velocity proportional to  $e^{ipt}$ , which is oscillatory at the roll frequency and produces no net dispersion for steady, nonzero roll rate. The residue of the pole at the origin remains unchanged from Eq. (20), except for a constant of proportionality. Hence, the feedbacks defined by Eqs. (24) and (25) will give zero dispersion.

### D. Open-Loop Response

We can obtain the open-loop response to impulsive and step trim moments by substituting the complex angle-of-attack results Eqs. (12) and (13) or (27) and (28) into Eq. (7) with the feedbacks set equal to zero. For the impulsive trim moment, the net dispersion velocity is

$$\Delta V_{\text{impulse}} = -\frac{iL_0 m_*}{m^2}$$
 (29)

which is independent of the damping parameter  $\nu$ . The dispersion occurs in a direction that initially coincides with the negative body z-axis when the impulse occurs about the body y-axis at time t=0. The response to a trim step is

$$\Delta V_{\text{step}} = \frac{L_{\alpha} \Delta m}{mp\omega}$$
 (30)

which is also independent of the damping parameter. The dispersion for this case occurs in a direction that initially coincides with the positive body y-axis about which the trim moment step occurs at time t=0. This open-loop result has been derived previously and shows the inverse dependence of dispersion on roll rate, whereas dispersion from an impulsive moment is independent of roll rate. The units in Eqs. (29) and (30) are consistent, because  $m_*$  is a moment impulse divided by moment of inertia, which has units of sec<sup>-1</sup>, whereas  $\Delta m_*$  is moment divided by moment of inertia, which has units of  $\sec^{-2}$ .

#### E. Optimal Control

A performance index of a system to control ballistic reentry-vehicle dispersion should include, in addition to dispersion, a measure of the control moments required to effect the control. Therefore, a performance index I is defined as

$$I = W_{1} \Delta V \Delta V^{*} + W_{2} \int_{0}^{\infty} m_{\delta_{y}}^{2} dt + W_{3} \int_{0}^{\infty} m_{\delta_{z}}^{2} dt$$
 (31)

where  $\Delta V^*$  is the complex conjugate of the dispersion velocity  $\Delta V$ ,  $m_{\delta}$  and  $m_{\Lambda}$  are the control moments defined by Eqs. (8) and (9), and  $W_{1}$ ,  $W_{2}$ , and  $W_{3}^{Z}$  are weighting constants. Because the classical method used to solve for dispersion velocity yields a solution for the transformed values of the control moments in the complex s-plane, the integral-square values of these moments in Eq. (31) can be readily obtained by the use of Phillips integrals. For example, if the control moment  $m_{\delta}$  (s) obtained from Eq. (9) with the complex angle of attack components for an impulsive trim moment, Eqs. (12) and (13), has the form

$$m_{\delta_{\mathbf{z}}}(s) = \frac{B_{3}s^{3} + B_{2}s^{2} + B_{1}s + B_{0}}{A_{4}s^{4} + A_{3}s^{3} + A_{2}s^{2} + A_{1}s + A_{0}}$$
(32)

then the integral square value of  $m_z$  in Eq. (31), defined as  $I_4$ , has the value

$$I_4 = N_4/D_4 \tag{33}$$

where

$$N_{4} = B_{3}^{2} \left( A_{0} A_{1} A_{2} - A_{0}^{2} A_{3} \right) + \left( B_{2}^{2} - 2 B_{1} B_{3} \right) A_{0}^{\prime} A_{1} A_{4}$$

$$+ \left( B_{1}^{2} - 2 B_{0} B_{2} \right) A_{0}^{\prime} A_{3}^{\prime} A_{4}$$

$$+ B_{0}^{2} \left( A_{2}^{\prime} A_{3}^{\prime} A_{4} - A_{1}^{\prime} A_{4}^{2} \right)$$

$$D_{4} = 2 A_{0}^{\prime} A_{4} \left( A_{1}^{\prime} A_{2}^{\prime} A_{3} - A_{0}^{\prime} A_{3}^{2} - A_{1}^{2} A_{4} \right)$$

The coefficients  $A_i$  are defined in terms of the feedback gains in Eqs. (15) through (18), with  $A_4 = 1$  and a = b = c = d = 0; the coefficients  $B_i$  are defined by

$$B_0 = (w^2 - p^2)^2 a_1 + vpb_1 \tag{34}$$

$$B_1 = va_1 + 2pb_1 + (w^2 - p^2) c_1 + vpd_1$$
 (35)

$$B_2 = a_1 + vc_1 + 2pd_1 \tag{36}$$

$$B_3 = c_1 \tag{37}$$

To achieve optimum control, values of the feedbacks must then be found that minimize the control moment integral  $I_4$  while limiting the dispersion velocity  $\Delta V$  to an allowable level.

### F. Numerical Examples

Open- and closed-loop responses to three different forms of disturbance moments were obtained from numerical integration of the equations of motion. The three moment forms are as follows: (1) a moment impulse, (2) a moment step, and (3) a finite duration pulse that consists of stepping the moment of form (2) in the opposite direction after a finite time delay. The missile dynamics characteristics used in the simulations and shown in Table 1 are representative of a long-range ballistic reentry vehicle in the region of peak reentry dynamic pressure. Identical control moments that consist of the attitude and rate feedbacks defined in Eqs. (9), (24), and (25) were used in all three cases.

Table 1. Vehicle Dynamics Characteristics

m = 2.57 slugs

s = 0.545 ft<sup>2</sup>

μ = 0.0448

ω = 196 rad/sec

L<sub>α</sub> = 1.10 × 10<sup>5</sup> lb/rad

ν = 5.19 sec<sup>-1</sup>

C<sub>L</sub>\* = 2.66 sec<sup>-1</sup>

p = 9.95 rad/sec

u = 16.18 kft/sec

The open- and closed-loop responses to the moment impulse are shown in Figs. 1 and 2, respectively. The impulse was approximated by a rectangular pulse 0.002 sec in duration (which is short compared with the period of the highest vehicle natural frequency) and of sufficient magnitude to give a peak open-loop angle-of-attack oscillation amplitude of approximately 1 degree, as shown in Fig. 1A. The open-loop transverse velocity components resulting from the impulse are shown in Figs. 1B and 1C, and the closed-loop vehicle response, for comparison, is shown in Fig. 2. The closed-loop angle-of-attack response is not significantly changed, but the mean value of the transverse velocity is reduced from approximately 3.5 ft/sec to effectively zero, as the theory predicts.

The response to a moment step is shown in Figs. 3 and 4. The missile is initially trimmed at near zero angle of attack, and a trim moment equivalent to 0.5 deg trim angle of attack is applied suddenly at time t = 0 and sustained. The open- and closed-loop angle-of-attack responses are shown in Figs. 3A and 4A, and corresponding transverse velocity cross plots are shown in Figs. 3B and 4B. The open-loop dispersion velocity is approximately 38 ft/sec whereas the closed-loop velocity has a near-zero mean.

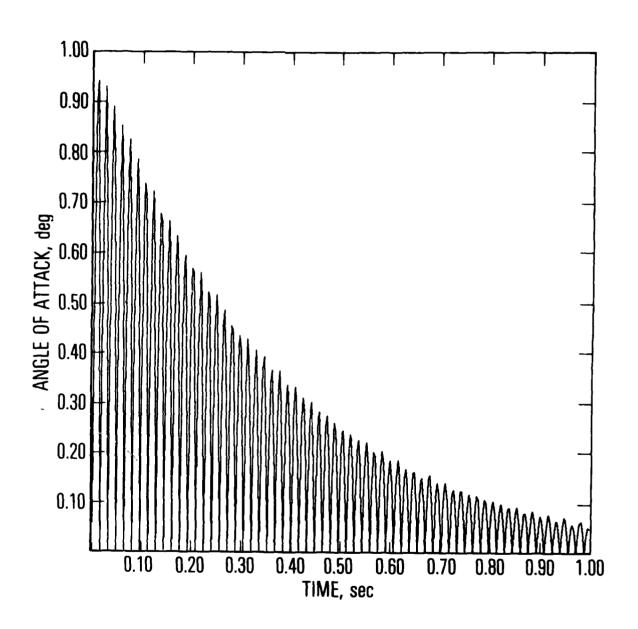


Figure 1A. Open-Loop Response to Moment Impulse

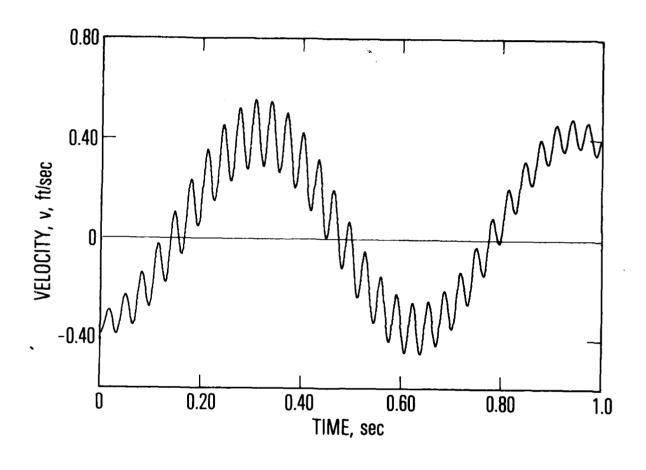


Figure 1B. Open-Loop Response to Moment Impulse

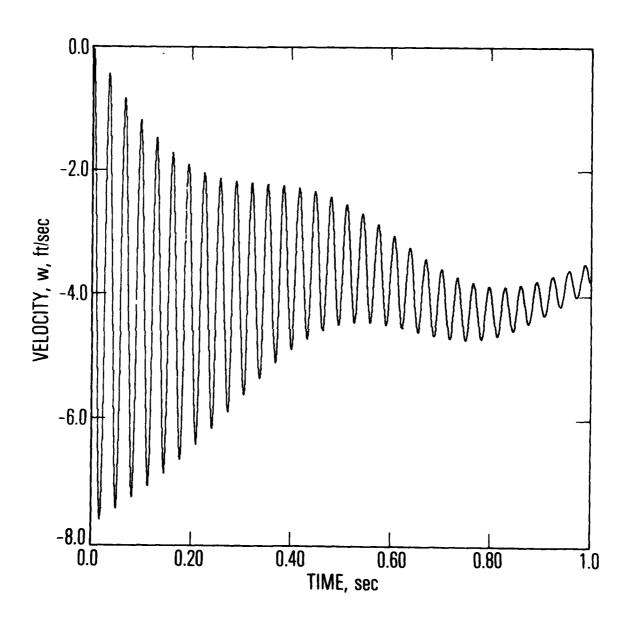


Figure 1C. Open-Loop Response to Moment Impulse

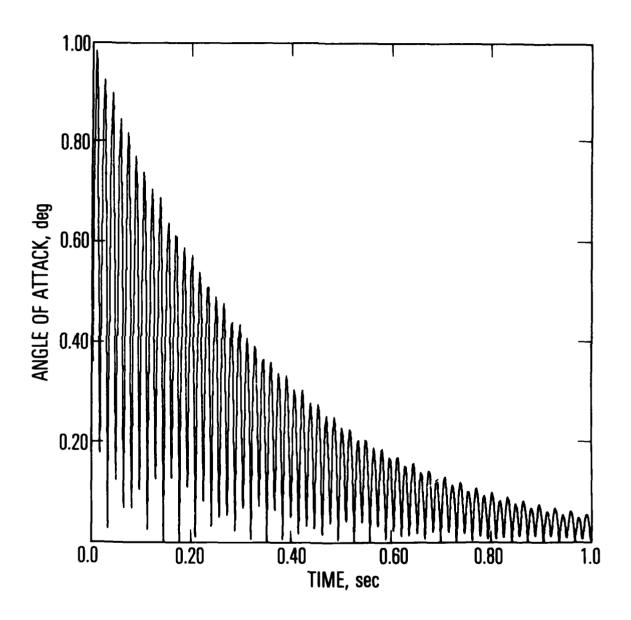


Figure 2A. Closed-Loop Response to Moment Impulse

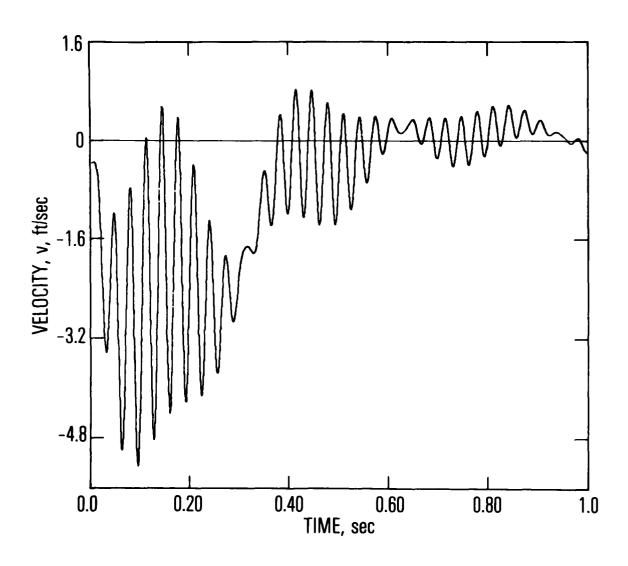


Figure 2B. Closed-Loop Response to Moment Impulse

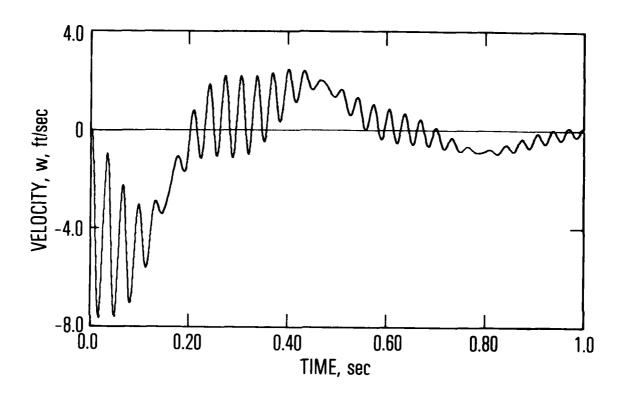


Figure 2C. Closed-Loop Response to Moment Impulse

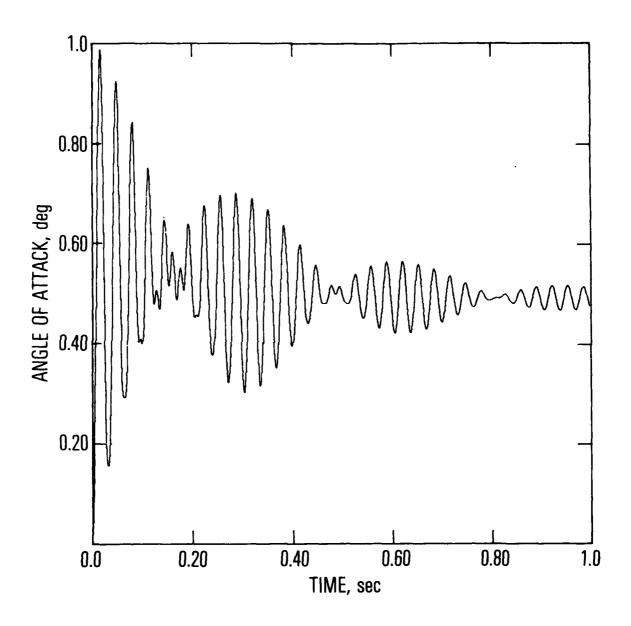


Figure 3A. Open-Loop Response to Moment Step

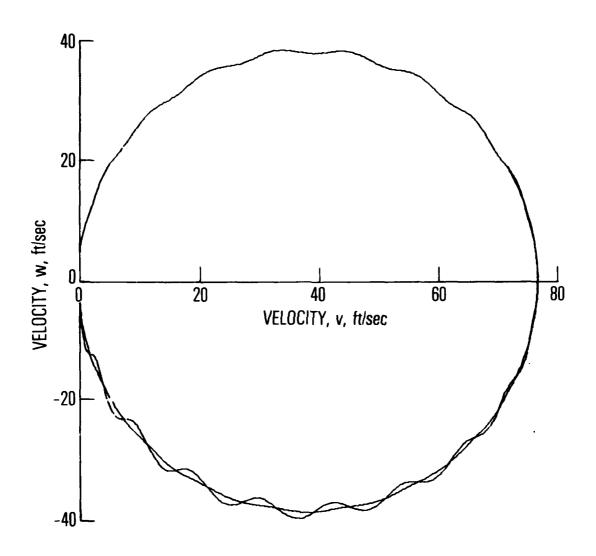


Figure 3B. Open-Loop Response to Moment Step

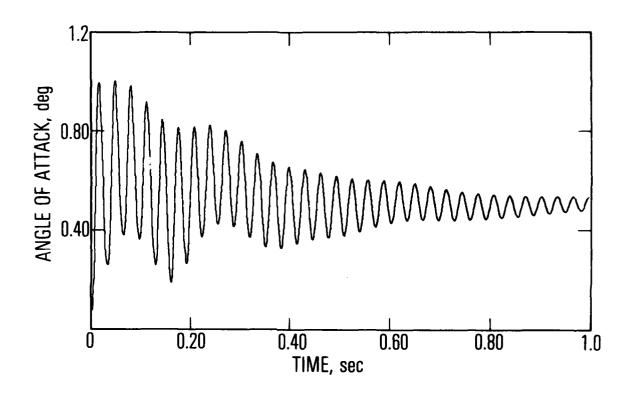


Figure 4A. Closed-Loop Response to Moment Step

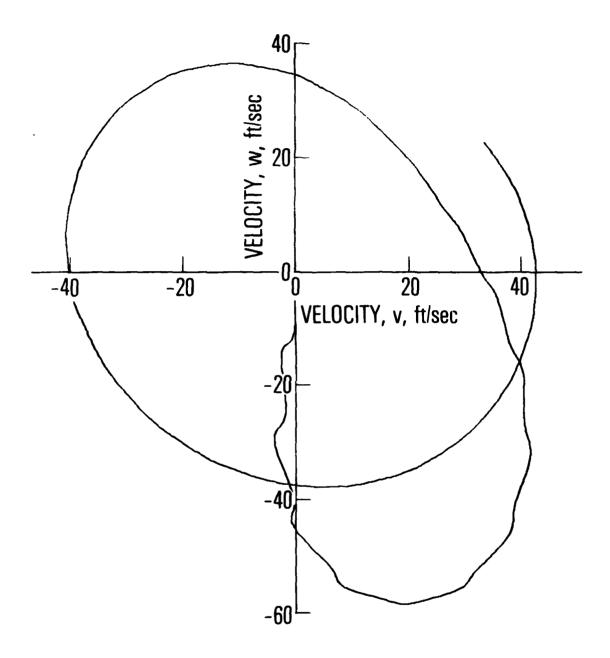


Figure 4B. Closed-Loop Response to Moment Step

The response to a moment pulse in which the moment in the example of Figs. 3 and 4 is removed at 0.5 sec is shown in Figs. 5 and 6. The net open-loop transverse velocity, obtained from the resultant of the velocity components of Fig. 5B, is approximately 46 ft/sec, while the closed-loop value from Fig. 6B is approximately 4 ft/sec.

The foregoing results were obtained with the feedback gains  $a_1$  and  $b_1$ , defined in Eqs. (24) and (25), that, in theory, cause zero dispersion. No consideration was given to optimization of the control moments. If the rate feedback gains  $c_1$  and  $d_1$  are included in addition to  $a_1$  and  $b_1$ , we can examine the influence of  $c_1$  and  $d_1$  on the integral square values of the control moments. The performance index I defined in Eq. (31) was evaluated numerically with different weighting constants to obtain feedback gains that produce minima in the integral square control moments. The results are given in Table 2 where the dispersion index  $\Delta V \Delta V^*$  is shown relative to the open-loop value, and the integral square control moment is shown relative to its value with  $c_1 = d_1 = 0$ .

#### G. System Implementation

The dispersion velocity to be controlled, as defined in Eq. (5), is an integral of the lateral acceleration in the two orthogonal cross-range inertial directions. This velocity was derived from the lift force expressed as a product of the lift force derivative  $L_{Q}$  (assumed to be constant) and the complex angle of attack 5. Without loss of generality, the quantity  $L_{Q}$ 5/m, which is the complex lateral acceleration, can be treated as the control parameter in place of angle of attack, because it can be measured directly with orthogonal lateral accelerameters. The control moments are defined in terms of feedback gains in lateral acceleration and acceleration rate, instead of angle of attack and angle-of-attack rate as defined in Eqs. (8) and (9). The acceleration rates can be obtained either by differentiation of the accelerometer outputs or from rate gyro measurements by use of the relation

$$\eta = q + ir = -i \left[ \dot{\xi} + \left( C_{L_{\alpha}}^* + ip \right) \xi \right]$$
(38)

between the complex lateral rate  $\P$  and complex angle of attack  $\P$ . If we define a complex normal acceleration

$$A_{N} = A_{v} + i A_{z}$$
 (39)

then the acceleration rate  $\dot{A}_N$  from Eq. (38) is

$$\dot{A}_{N} = \frac{iL_{\alpha}}{m} (q + ir) - \left(C_{L_{\alpha}}^{*} + ip\right) A_{N}$$
 (40)

The control moments are generated from small pitch and yaw trim flaps or reaction jets to produce a net control moment in a plane determined from a resolution of the lateral accelerometer measurements. The control system acts as a regulator to maintain a zero value for the net transverse dispersion velocity.

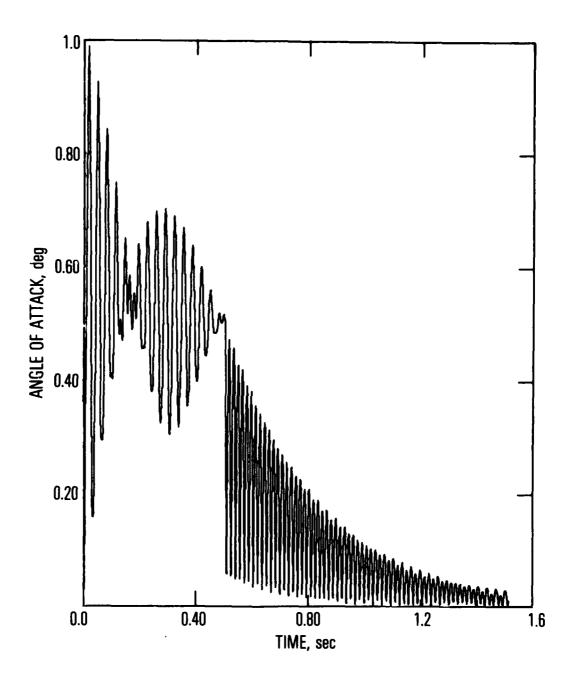


Figure 5A. Open-Loop Response to Moment Pulse

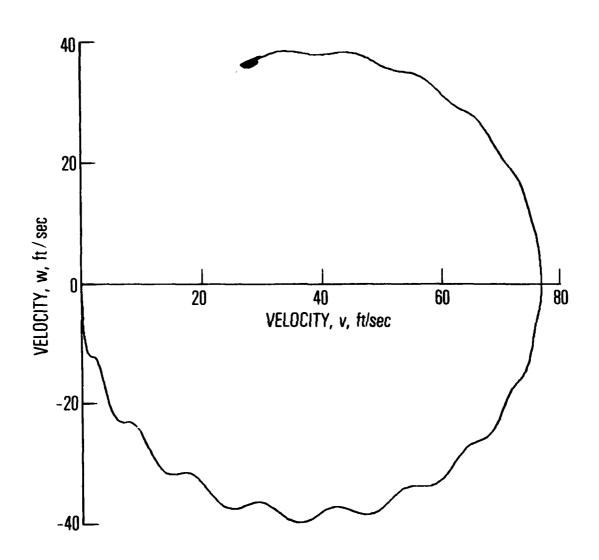


Figure 5B. Open-Loop Response to Moment Pulse

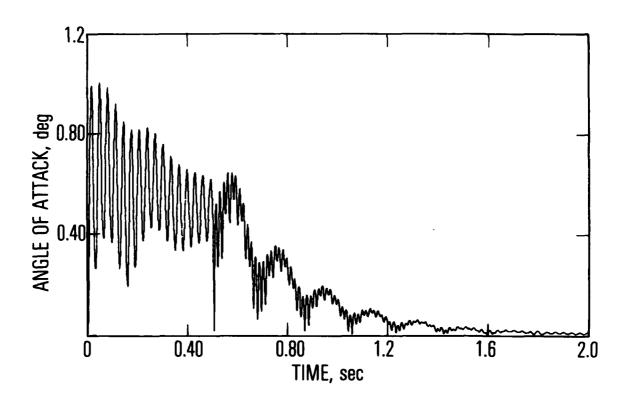


Figure 6A. Closed-Loop Response to Moment Pulse

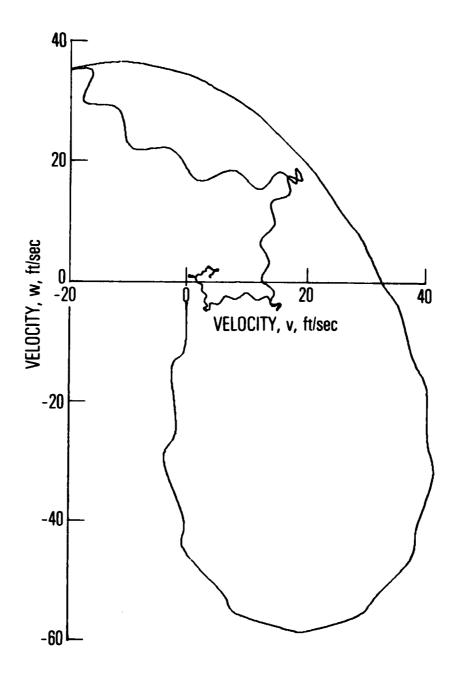


Figure 6B. Closed-Loop Response to Moment Pulse

Table 2. Influence of Feedback Gains on Performance

	Feedback Gains	ains		Dispersion Index,	
a	b <sub>1</sub>	$c_1$	d <sub>1</sub>	(AVAV )/(AVAV )O. L.	Index, 1 <sub>4</sub> /1 <sub>4</sub>
0	0	0	0	1.0	
1689	-36,570	-44.3	592	0.217	0.366
1946	-37, 230	-45.3	216	0.0365	0.435
103	-38, 380	0	0	0~	1.0

#### III. SUMMARY AND CONCLUSIONS

A control system has been formulated for limiting cross-range dispersion of a spinning reentry vehicle caused by lift nonaveraging. The system would use small trim flaps or reaction jets to minimize transverse dispersion velocity based on information derived from body-fixed lateral accelerometers and rate gyros. A simple control law derived for impulsive-type disturbance moments indicates that a fixed set of feedbacks will effectively limit dispersion for different generic moment forms that can otherwise produce large dispersion. For a special condition in which the roll rate is maintained at 50% of the vehicle natural pitch frequency, control can be achieved with information derived entirely from lateral accelerometers. Open- and closed-loop vehicle responses have been demonstrated with digital computer simulations of the equations of motion.

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## ABBREVIATIONS AND SYMBOLS

a,b, c, d	feedback gains
a <sub>1</sub> , b <sub>1</sub> , c <sub>1</sub> , d <sub>1</sub>	feedback gains
A <sub>0</sub> , A <sub>1</sub> , A <sub>2</sub> , A <sub>3</sub>	coefficients of characteristic equation
A <sub>N</sub>	complex normal acceleration, $A_y + iA_z$
Ay	y-component of normal acceleration
$\mathbf{A}_{\mathbf{z}}$	z-component of normal acceleration
$c_{\mathtt{L}_{\pmb{lpha}}}$	lift force derivative
$^{C}\dot{\mathbb{L}}_{\pmb{lpha}}$	damping parameter, $C_{L_{lpha}}^{}$ QS/mu
C* m	pitch damping parameter, -C <sub>m</sub> QSd <sup>2</sup> /2Iu
d	reference diameter
F	Eq. (3)
G	Eq. (4)
H(t)	Heaviside unit step function
i	$\sqrt{-1}$
I	pitch or yaw moment of inertia
I <sub>x</sub>	roll moment of inertia
L <sub>α</sub>	lift force derivative, $C_{\mathbf{L}_{\pmb{lpha}}}^{\ \ QS}$
m	vehicle mass
$\mathbf{m_t}$	complex trim disturbance moment, $m_{t_y} + im_{t_z}$
m <sub>8</sub>	complex control moment, $m_{\delta_y} + im_{\delta_z}$
m <sub>*</sub>	moment impulse
Δm <sub>y</sub>	moment step
N <sub>0</sub> , N <sub>1</sub> , N <sub>2</sub>	coefficients of numerator polynomial

p	roll rate
q	pitch rate
Q	dynamic pressure
r	yaw rate
S	Laplace transform variable
S	aerodynamic reference area (base area)
t	time
u	vehicle velocity
v	component of transverse dispersion velocity
$\Delta V$	complex transverse dispersion velocity, v + iw
w	component of transverse dispersion velocity
α	angle of attack
β	angle of sideslip
δ( <b>t</b> )	unit impulse function
Δ	characteristic, Eq. (14)
η	complex lateral rate, q + ir
μ	moment of inertia ratio, Ix/I
V	aerodynamic damping parameter, $C_{m_q}^* + C_{N_a}^*$
5	complex angle of attack, $\beta + i\alpha$
ω	undamped natural pitch frequency

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